647510 **MSC1MatC104x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Jan/Feb.-2022 [NEW COURSE]
Theory of Ordinary Differential Equations(Core (New))

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q-1 (A) Answer any Two out of Three **[10]**

(1) Discuss the Abel's formula for the first order linear differential equation.

(2) Solve the following initial value problem $y_1{}' = y_1 + y_2 + f(t)$ and $y_2{}' = y_1 + y_2$ with the initial conditions $y_1(t_0) = 0,\ y_2(t_0) = 0$. where f is continuous from I to R, $t_0 \in I$.

(3) Prove that the Wronskian of the solutions of the homogeneous system is either identically zero or nowhere zero on finite interval.

Q-1(B) Answer any One out of Two **[4]**

(1) Find the general solution of the following system of differential equations. $\frac{dx}{dt} = x + y$ and $\frac{dy}{dt} = 4x - 2y$.

(2) Show that $\sin t^2$ and $\cos t^2$ are linearly independent solutions of $ty'' - y' + 4t^3y = 0$ on $(-\infty, 0)$ and $(0, \infty)$.

Q-2 (A) Answer any Two out of Three **[10]**

(1) Show that Eigen vectors corresponding to the distinct Eigen values of square matrix are linearly independent.

(2) Let A be any constant matrix of order 2 with complex eigen value $\alpha \pm i\beta$. Then prove that there exist a non singular matrix T of order 2 such that $T^{-1}AT = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}$.

(3) Find the solution of $y' = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} y + \begin{bmatrix} e^t \\ 1 \end{bmatrix},\ y(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Q-2 (B) Answer any One out of Two **[4]**

(1) Find the fundamental matrix of $y' = Ay$ on R and obtain $\exp(tA)$ on R. where $A = \begin{bmatrix} 3 & 5 \\ -5 & 3 \end{bmatrix}$.

(2) Find the Eigen values and corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}$

Q-3 (A) Answer any Two out of Three **[10]**

(1) Solve $y'' - xy' - y = 0$ near zero.

(2) State and prove Fourier Bessel Expansion theorem.

(3) Prove that $e^{\frac{x}{2}(t - \frac{1}{t})} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)[t^n + (-1)^n t^{-n}]$

Q-3 (B) Answer any One out of Two **[4]**

(1) Classify singularities of the differential equation

$$x(x-1)^2(x+2)y'' + x^2(x-1)y' + (x+2)y = 0 .$$

(2) Find the derivative of $x^{\alpha}J_{\alpha}(x)$ with respect to x.

Q-4 (A) Answer any Two out of Three **[10]**

(1) State and prove existence and uniqueness theorem for system of first order differential equations.

(2) Discuss the variation of constant formula for unique solution of second order linear non homogenous differential equation.

(3) Find the particular solution of $y'' + y = \tan t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Q-4 (B) Answer any One out of Two **[4]**

(1) Determine whether the following initial value problem have a solution or not. If yes, find the interval of existence of solution.

$$(t^2+4)y'' + ty' + y\sin t = 1, \quad y(1) = 2,\ y'(1) = 0.$$

(2) Find out whether the following system has a solution or not.

$$y_1{}' = ty_2 + y_3, \quad y_2{}' = y_1 \cos t + t^2 y_3, \qquad y_3{}' = y_1 - y_2 .$$

Q-5 (A) Answer any Two out of Three **[10]**

(1) Using the Laplace transforms, find the solution of the differential equation $y'' - 4y' + 4y = 64\sin 2t,\ y(0) = 0, y'(0) = 1$.

(2) Find the Laplace transform of $\frac{1-\cos t}{t^2}$.

(3) State and prove convolution theorem.

Q-5 (B) Answer any One out of Two **[4]**

(1) Find the Laplace transforms of $t^2 e^t \sin 4t$.

(2) Find the inverse Laplace transforms of $\frac{s+4}{s(s-1)(s^2+4)}$.

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